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The flow and heat transfer in the wedge-shaped liquid film formed during the growth of a vapour bubble

J. MITROVIC†

Institut für Technische Thermodynamik und Thermische Verfahrenstechnik, Universität Stuttgart, 70550 Stuttgart, Germany

Abstract—After providing simple equations that should govern the behaviour of a wedge-shaped liquid film sandwiched between the vapour in a growing bubble and the heated wall, we discuss some questions that may be important for modelling heat transfer in nucleate boiling. Considered are the curvature of the film surface and the liquid flow in the film, for example. We further examine the consequences of some simplifications adopted in models from the literature. Our analysis indicates that the bubble interface is concave–convex in the region of the three-phase-line (TPL). The convex curvature is caused by strong evaporation at the tip of the liquid wedge under no slip condition of the liquid on the wall surface. The ultra-thin layer of this wedge adhering to the wall is practically motionless and hence incapable of supplying the quantity of liquid to the film tip necessary for evaporation to occur rapidly at a fixed TPL. Based on this fact, we are led to the conclusion that, under common boiling conditions, the TPL and the film interface are not motionless, as was assumed in very recent models of bubble growth. These models are shown in the present paper to be inconsistent. Namely, the liquid flow orthogonal to the heated surface is ignored within the hydrodynamics, but this flow is indeed included into the energy balance of the models. © 1998 Elsevier Science Ltd. All rights reserved

1. INTRODUCTION

To explain some features observed in colloid systems, such as adhesion and coagulation of particles, Kallmann and Willstaeter [1] proposed in 1932 a theory which is based upon the notion of an attractive and a repulsive force acting between the particles. According to this theory, the repulsive force has its origin in electrostatic charges of the interfaces and should decrease as $\exp(-m\delta)$, while the attractive force should change according to δ^{-n}, if δ is the distance between the particles, that is, the thickness of the liquid film sandwiched between them; m and n are adjusting parameters. The condition of equal magnitudes of the two forces determines the equilibrium position of the particles relative to one another.

Relatively soon after the publication of Kallmann and Willstaetter, in 1934, Derjaguin and his colleagues (for review see ref. [2]) reported on the state and properties of thin liquid films separating two identical, or different, phases. They showed that the trapped layer of a liquid, formed when a gas bubble is pressed against a solid plate immersed in the liquid, slowly

thins and leaves under isothermal conditions a uniform equilibrium film. The pressure in the film has been found to be different from the pressure in the liquid bulk, which communicates with the film. The difference of these pressures is termed the 'disjoining pressure', probably because of its action that opposes the film thinning and hence hinders the film interfaces from joining each other.

Over the past two decades, the concept of the disjoining pressure has been more and more involved in modelling the flow in thin liquid films which may play an important role in several branches of thermal systems, such as for example heat pipes. As the author of this paper is aware, the idea to couple the disjoining pressure and liquid flow in the film was introduced by Nerpin and Derjaguin [3] in 1955, and has meanwhile been refined and extended by several authors in a number of papers. Some twenty years later, seemingly for the first time, Wayner et al. [4] considered the heat transfer in a thin film, driven simultaneously by gradients of disjoining and capillary pressures. For a limited literature review concerning the liquid menisci which can be expected to form in heat pipes, the reader may be referred to a monograph by Faghri [5]. The main aim of the studies quoted there was to predict the heat transfer associated with evaporation in the region of a stationary, motionless three-phase-line (TPL) formed in grooves of the heating zone of heat pipes.‡ The papers by DasGupta et al. [6], Ha and

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† Tel.: 0049 711 685 6013. Fax: 0049 711 685 6140. E-mail: mitrovic@itt.uni-stuttgart.de.

‡ The term 'three-phase-line' (TPL) is used in this paper to characterise the region where all the phases (liquid, vapour, solid) act on each other, even if the vapour phase is separated from the wall by an adsorbed layer.

NOMENCLATURE

A_0	Hamaker constant		η	axial coordinate
g	acceleration due to gravity		κ	curvature
$\dot{H}$	enthalpy flow		ν	kinematic viscosity
$\dot{M}$	mass flow		ξ	radial coordinate
p	pressure		ρ	density.
q	heat flux			
$\dot{Q}$	heat flow			
TPL	three-phase-line		Subscripts	
u	velocity.		I	interface
			l	liquid
Greek symbols			TPL	at TPL
δ	film thickness		δ	at film surface
Δh	latent heat		η	in η-direction
Γ	flow rate		ξ	in ξ-direction.

Peterson [7], and Reyes and Wayner [8] contain further details about the subject.

The results achieved with evaporation at stationary liquid menisci were apparently so promising that the thin-film-model has recently been applied—without substantial alteration—to describe the fluid flow and heat transfer associated with bubble growth by Lay and Dhir [9]. A liquid film is assumed to exist between the vapour in the bubble and the heated wall. In the model, actually, the bubble (vapour mushroom) is connected to the heated surface by several vapour stems, and each stem is separated from the heated surface by a thin liquid film. The inner region of the film is considered as an adsorbed layer incapable of evaporation, while the outer region is the evaporating film; its surface is only concave. The film interface is motionless, the evaporating liquid is steadily delivered by flow in the film radially inwards. This flow is driven by gradients of capillary and disjoining pressures.

Basically the same model was developed independently by Hammer and Stephan [10–13]. The vapour stems, assumed in the model by Lay and Dhir, are replaced by a continuous liquid film, sandwiched between the vapour in the bubble and the heated surface. The inner region of the film is non-evaporating, while its outer region is vapour-generating. The position of the film interface is considered to change jumpwise radially outwards, as the bubble grows. Between any two subsequent jumps, the interface remains fixed in space. This necessitates the liquid to flow radially inwards, if evaporation at the film surface should take place. All transport processes occurring are assumed to be steady and time-independent. The numerical results obtained from the model are compared with some experimental data of heat transfer studies and the agreement between them is found to be exceedingly good. The deviations are reported to lie still within the experimental uncertainty even in nucleate boiling on a horizontal tube [14], where the self induced external liquid flow, non-symmetry of the film underneath the growing bubble, and sliding of detached bubble around the tube circumference influence the heat transfer in a complex manner.

It should be noticed, however, that the validation of the models was—and still is—not possible at the microscopic level of the basic idea. Such a validation, important and desirable as it is, is precluded yet. Due to accidental nature of bubble appearance, the micro-scale of the system, and the rapidity of the processes occurring instantly, direct measurements in the region of the TPL are scarcely possible, and they seem to have not been performed up to now. For this reason, the authors [9–13] had to consider and to model further quantities and processes associated with them, such as, for example, the density of the actually active bubble formation centres on the heated surface and the contribution of convection to the overall heat transfer. Needless to say that uncertainties introduced by modelling of these processes overshadow to some extent the basic idea of film evaporation and tarnish its performances.

Over two decades ago, probably for the first time, the author of the present paper emphasised that the heat transfer and energy transformation, associated with bubble growth, are concentrated along the TPL [15], where the rapid evaporation leads to a concave–convex curvature of the film surface, Fig. 1. This generates and maintains the Laplace pressure, whose action, combined with the evaporation taking place at TPL, is considered to be responsible for the increase of the area across which the vapour in the bubble interacts with the heated wall. With regard to heat transfer, the TPL is seen as a circular, line-like heat sink, expanding at first radially outwards. After a certain period of time, the movement of the TPL is
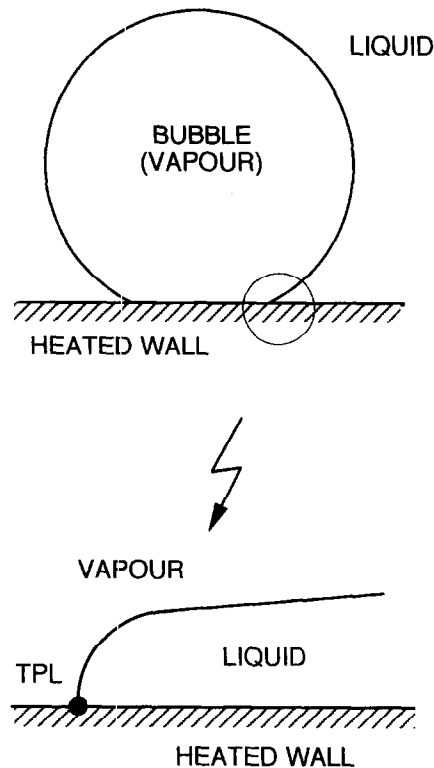
Fig. 1. Vapour bubble growing on a heated surface. The detail illustrates the concave–convex curvature of the interface caused by evaporation along the TPL ('intersection' of bubble interface with the heated surface).

reversed. This initiates the bubble detachment process, during which the TPL constricts until the concave radius of the bubble neck disappears and the bubble takes-off.

The papers [9–13] ignore the origin of the idea that associates the rapid evaporation at the tip of the liquid wedge with bubble growth. The evaporation at the TPL, discussed by the present author in a further paper [16], has been dismissed in ref. [10] summarily with the remark†

"The terms, such as dry spot or three-phase-line, used in the literature, are meaningless for the present model, because at no place does the vapour phase touch the wall".

Regarding this model assumption, we should remember that bubble growth is only one event—undoubtedly, an important one—in the chain of events making up the bubble cycle, and the question, whether or not the three-phase-line does indeed exist can hardly be answered alone on the basis of a relatively narrow notion about heat transfer. For a satisfy-

† "In der Literatur gebräuchliche Begriffe, wie trockener Fleck oder Tripellinie sind in diesem Modell bedeutungslos, da an keiner Stelle die Dampfphase mit der Wand in Berührung kommt", p. 7 in ref. [10].

ing treatment of this question, a thorough study would be required, including also processes of essential importance for phase change. In contrast to the above statement, a supposition of such a line to exist seems realistic for several reasons. For example, if the vapour actually does not at all interact with the heated surface, as the statement claims, the adsorption layer, admitted in the model, must have been established prior to bubble generation, and the processes of bubble nucleation and bubble growth are not able to destroy it. On the assumption that the statement is actually true, we are faced with serious difficulties, when we are to attempt to explain the nucleation process. In such a case, the bubble nucleus must originate and be activated somewhere in the liquid outside from the adsorption layer, which means, under conditions comparable with those of homogeneous nucleation. Homogeneous nucleation, however, is known to require liquid superheats much larger than those observed in common boiling.

Moreover, also the state of the layer, that might be adsorbed on the heated surface and separate the vapour in the bubble from the wall, has not been investigated and is hardly understood yet. Heated surfaces, used in boiling experiments, are heterogeneous and show distinct centres regarding adsorption. Between such centres, a two-dimensional movement of molecules, a surface diffusion, is likely to occur. The state of these molecules is neither gaseous nor liquid; their thermal agitation orthogonal to the heated surface is surely reduced, but their surface diffusion and the interaction with neighbouring molecules can basically rupture the eventually closed adsorption layer, particularly at tips of roughness elements of nano-range. The above statement taken from ref. [10] seems thus to be insufficient even for an oversimplified picture of the physical reality. A detailed analyses of adsorption, including specific features of the case considered, amount and surface activity of gases dissolved in the liquid, and topology and micro-relief of the heated surface, seems unavoidable if the mutual interaction between vapour and heated wall across the area of their contact should be investigated a little more closely.

Furthermore, it is worth mentioning that we, even on the basis of the above statement, are able to construct an 'effective' TPL. Since the model admits the existence of an adsorbed layer, which is motionless and incapable of evaporation, and which must be formed prior to bubble generation, we may consider this layer to be 'rigid', thus representing the outermost layer of the heated wall. The heated surface may, therefore, be thought as being shifted outwardly and fixed at the outer edge of the adsorption layer. In this limiting case, we certainly may speak about a three-phase-line as far as evaporation is concerned.

Some motivation for the present paper lies in the simplifications adopted in the models [9–13], and first of all the ignoring of the axial flow (orthogonal to the heated surface) in the liquid film. These simplifications

are discussed in the paper and their consequences pathwise quantified. We further exploit relationships based on elementary balances and show that the interface at the TPL of a growing bubble is concave–convex. A comparison of the expression for the heat flux derived in this paper with the corresponding one used in the models leads us to an inconsistency of these models. This inconsistency concerns the axial flow velocity in the liquid film which is ignored within the film hydrodynamics, but included into the relationship for the heat flux. The questions raised concern not only the case of nucleate boiling. They are, at least partly, addressed also to evaporation at liquid menisci in general.

The remainder of the paper is organised as follows: in the next two sections, the hydrodynamics of a wedge-shaped liquid film is briefly treated; the heat flux is connected with the main flow properties. The equations derived are more or less already in use, and these sections are, therefore, largely a summary of what is known about the film flow so far adopted for modelling. These equations, together with the simplifications that made their derivations possible, are then examined in the last section.

The following assumptions are considered to be confirmed:

—single component fluid of constant physical properties,
—steady-state and laminar liquid flow,
—ideally smooth and plane heated surface which is arranged horizontally.

Almost all considerations in this article are quite elementary. This simple material may nevertheless be helpful to some reader trying to model the nucleate boiling.

2. FLOW IN A WEDGE-SHAPED LIQUID FILM

2.1. The governing equations

In this section, the constitutive equations for the film flow are derived from the Navier–Stokes equations, adopting simplifications as usual in the literature.

Figure 2(a) illustrates a part of a wedge-shaped liquid film sandwiched between the vapour in a bubble and an ideally smooth, heated surface. Similar illustrations can be taken from several sources, e.g. ref. [11]. The vapour–liquid interface is assumed to be fixed in space, the evaporation at the film surface is considered to be stationary. The vapour leaving the interface is thus steadily replenished by liquid flow in the film, the flow itself is strictly viscous.

Using the cylindrical coordinate system indicated in Fig. 2(b), in which η is the axial coordinate, and at the same time the symmetry axis of the bubble, the steady-state momentum equations and the equation of continuity can be written as
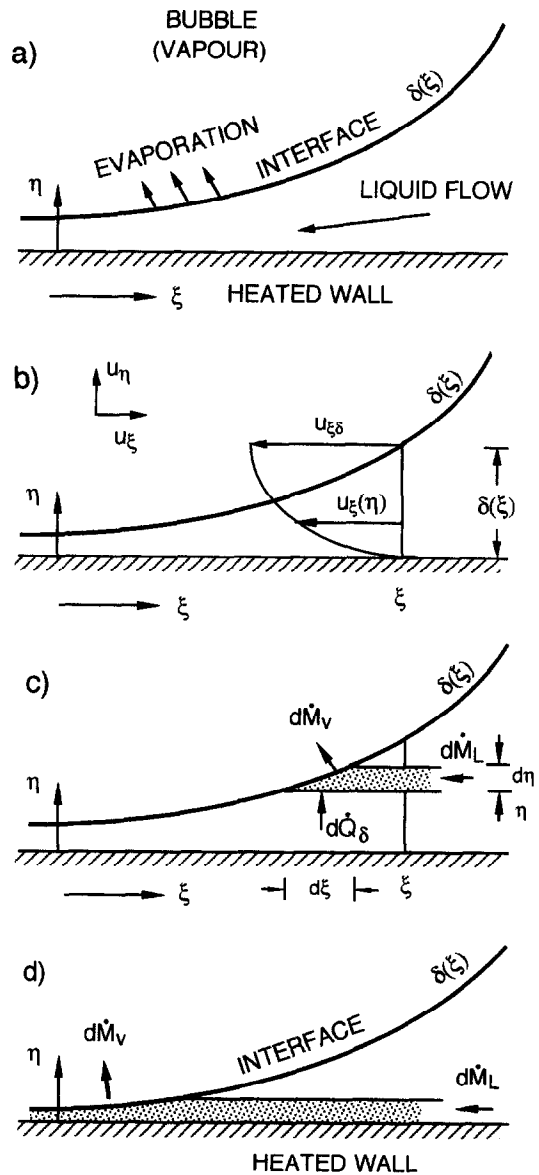

Fig. 2. Illustrations representing the thin-film-model of heat transfer associated with bubble growth. (a) General impressions of liquid flow and evaporation at the interface as adopted in ref. [9–13]. (b) Schematic of velocity distribution in the cylindrical coordinates; the axial coordinate η coincides with the symmetry axis of the bubble. In the models, the velocity u_η is ignored. (c) Elementary 'flow channel' (shaded area) illustrating the constancy of liquid flow. Since $u_\eta = 0$, the mass flow $d\dot{M}_1$ does not change along the flow path. (d) The same flow channel as in (c), now located directly on the heated surface. Due to proximity of the surface, the mass flow $d\dot{M}_1$ in this channel is very low, but the heat flux at the 'outlet' of the channel is very large.

$$u_\xi \frac{\partial u_\xi}{\partial \xi} + u_\eta \frac{\partial u_\xi}{\partial \eta} = f_\xi - \frac{1}{\rho_1} \frac{\partial p_1}{\partial \xi}$$

$$- \nu_1 \left(\frac{\partial^2 u_\xi}{\partial \eta^2} + \frac{1}{\xi} \frac{\partial u_\xi}{\partial \xi} - \frac{u_\xi}{\xi^2} + \frac{\partial^2 u_\xi}{\partial \xi^2} \right) \quad (1)$$

$$u_\xi \frac{\partial u_\eta}{\partial \xi} + u_\eta \frac{\partial u_\eta}{\partial \eta} = f_\eta - \frac{1}{\rho_1} \frac{\partial p_1}{\partial \eta}$$

$$+ v_1 \left(\frac{\partial^2 u_\eta}{\partial \eta^2} + \frac{1}{\xi} \frac{\partial u_\eta}{\partial \xi} + \frac{\partial^2 u_\eta}{\partial \eta^2} \right) \quad (2)$$

$$\xi \frac{\partial u_\eta}{\partial \eta} + \frac{\partial}{\partial \xi} (u_\xi \xi) = 0 \quad (3)$$

where f, ρ, v and p are the body force, the density, the kinematic viscosity and the pressure, respectively. The index 1 refers to the liquid phase, the directions of the velocities u_ξ and u_η can be taken from the figure.

It should be noted that the third momentum equation does not appear in the above set of equations for the reason of a tacitly assumed rotational symmetry. Such an assumption is largely justified for a single bubble growing on a horizontal, perfectly smooth heated surface. However, for bubbles growing on a horizontal tube, or a vertical surface, this assumption must carefully be examined.

The simplification usually made when dealing with thin films states that

$$\xi \frac{\partial u_\eta}{\partial \eta} \ll \frac{\partial}{\partial \xi} (u_\xi \xi) \quad (4)$$

and the equation of continuity is written as

$$\frac{u_\xi}{\xi} = -\frac{\partial u_\xi}{\partial \xi}, \quad \text{or } \xi u_\xi = \text{const.} \quad (5)$$

This simplification is important for our further analyses. For this reason we should observe that the inequality (4) requires loosely either ξ or $\partial u_\eta / \partial \eta$ to be small, but equation (5) clearly states that $\xi \partial u_\eta / \partial \eta$ must be zero. Since ξ is arbitrary, the latter condition is fulfilled only at $\partial u_\eta / \partial \eta = 0$, which results in $u_\eta = \text{const} = 0$ for an impermeable wall surface. This simplifies the equations (1) and (2) considerably because the fluid is allowed to flow only radially.

By a further assumption, the term $u_\xi \partial u_\xi / \partial \xi$ on the left-hand side in equation (1) is omitted, either due to small velocity u_ξ itself, or its derivative.* Thus, for a horizontal wall arrangement ($f_\eta = -g$) without body force in the radial direction ($f_\xi = 0$), the equations (1) and (2) become

$$\frac{\partial p_1}{\partial \xi} = v_1 \rho_1 \left(\frac{\partial^2 u_\xi}{\partial \eta^2} + \frac{1}{\xi} \frac{\partial u_\xi}{\partial \xi} - \frac{u_\xi}{\xi^2} + \frac{\partial^2 u_\xi}{\partial \xi^2} \right) \quad (6)$$

$$\frac{\partial p_1}{\partial \eta} = -\rho_1 g. \quad (7)$$

The axial body force f_η adopted is constant. It arises from the action of gravity and is different from that in ref. [10], where $f_\eta \sim -1/\eta^4$ leads to a singularity as η tends to zero.

In equation (6), the sum of the last three terms in the

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* Using equation (5), we get $u_\xi \partial u_\xi / \partial \xi = \text{const}/\xi^3$ and the ignoring the term $u_\xi \partial u_\xi / \partial \xi$ can lead to serious errors particularly at small ξ. The errors will be larger the larger the distance from the surface, because the constant in this expression do actually depend on η, and it rises as η increases.

parenthesis is zero, as follows from a differentiation of equation (5). Equation (6) thus takes the form

$$\frac{\partial p_1}{\partial \xi} = v_1 \rho_1 \frac{\partial^2 u_\xi}{\partial \eta^2}. \quad (8)$$

The equations that describe the flow and pressure fields in the film and that have to be solved are the equations (7) and (8). An integration of equation (7) gives

$$p_1 = -\rho_1 g \eta + \varphi(\xi) \quad (9)$$

where the function of integration $\varphi(\xi)$ has to be determined either from the remaining momentum equation (8), or elsewhere, as sketched below.

Equation (9) shows that the simplification of hydrodynamic equations results in a linear change of the pressure p_1 in the film along the coordinate η. Along the vapour–liquid interface ($\eta = \delta$), the pressure distribution ($p_1 = p_{1\delta}$) is described by

$$p_{1\delta} = -\rho_1 g \delta + \varphi(\xi) \quad (10)$$

which, because of $\delta = \delta(\xi)$, is a function of ξ only.

Since the derivative $\partial p_1 / \partial \xi = \varphi'(\xi)$ is independent of η, as follows from equation (9), also equation (8) can immediately be integrated. The boundary conditions, no slip at the wall surface, $\eta = 0 : u_\xi = 0$, and no shear stress at the interface, $\eta = \delta : \partial u_\xi / \partial \eta = 0$, give then the following expression for the radial velocity component u_ξ:

$$u_\xi = \frac{1}{v_1 \rho_1} \left(\frac{\eta^2}{2} - \delta \eta \right) \frac{\partial p_1}{\partial \xi}. \quad (11)$$

The direction of the velocity u_ξ is determined by the sign of $\partial p_1 / \partial \xi$, since, because of $\eta < \delta$, the difference of the terms in the brackets in equation (11) is negative. For the flow to be directed radially inwards, as indicated in Fig. 2(b), the derivative $\partial p_1 / \partial \xi$ must therefore be positive.

A remark is needed regarding the boundary condition at the film surface ($\eta = \delta$) included into equation (11). This condition roots in the Nusselt pioneering paper on vapour condensation and has meanwhile been used by several authors in various physical problems. So in the case of growing bubble, without any limitation or trace of doubt about its validity. In this case, however, the condition is neither obvious (foremost at fast bubble growth) nor has it been thoroughly examined, but it is of an essential importance, because it introduces the function $\delta(\xi)$ into the final equation and influences the results not only quantitatively. We shall return to this question further below, when discussing the results in general.

2.2. Flow rate and pressure distribution

Equation (11) allows a calculation of the flow rate in the cross-sectional area $2\pi\xi\delta$ of the film. Denoting by Γ the total mass flow of liquid per unit of length

orthogonal to the coordinate ξ (circumference of the circle, $2\pi\xi$), then

$$\Gamma = \rho_1 \int_0^\delta u_\xi \, d\eta = -\frac{1}{3}\frac{\delta^3}{v_1}\frac{\partial p_1}{\partial \xi}. \qquad (12)$$

Combining this equation with equation (9), the following relationship for the function $\varphi(\xi)$ is arrived at

$$\varphi(\xi) = -3v_1 \int \frac{\Gamma}{\delta^3} \, d\xi + C \qquad (13)$$

giving

$$p_1 = -\rho_1 g\eta - 3v_1 \int \frac{\Gamma}{\delta^3} \, d\xi + C. \qquad (14)$$

Equation (14) connects the pressure p_1 in the film with the flow rate Γ and the film profile, represented by the film thickness δ. For the constant of integration to be determined, an apropriate boundary condition could be chosen.

The above system of equations requires specifications of further quantities for a solution of the hydrodynamic problem. Usually, the function $\varphi(\xi)$ is prescribed by

$$\varphi(\xi) = \frac{1}{6\pi}\frac{A_0}{\delta^n} + \kappa\sigma \qquad (15)$$

where A_0, κ and σ denote the Hamaker constant, the curvature of the interface and the surface tension, respectively. The first term in this equation is the disjoining pressure that arises from the interaction of the film with its substrate (wall), while the second term is the Laplace pressure. The curvature κ and the exponent n depend on the film thickness δ; the signs of the terms making up the function $\varphi(\xi)$ must be chosen in a way, that should lead to a pressure derivative necessary for the required direction of film flow, see equation (11). Details concerning equation (15) can be taken from several sources, e.g. ref. [17], reviews [18, 19], and particularly from the monograph by Derjaguin et al. [2], and need therefore not be given here. For present purpose, only a short note regarding the disjoining pressure shall be made further below.

From equations (13) and (15), the following equation is obtained for determination of the film thickness δ,

$$\frac{1}{6\pi}\frac{A_0}{\delta^n} + \kappa\sigma = -3v_1 \int \frac{\Gamma}{\delta^3} \, d\xi + C. \qquad (16)$$

In studies dealing with flow in thin liquid films, the hydrostatic contribution to the pressure in the film is, as a rule, neglected leading at $\xi = $ const. to a constant pressure over the whole film thickness δ. The pressure

in the film thus changes only with the radial position ξ (contained implicitly in δ and κ) according to

$$p_1 = \varphi(\xi) = \frac{1}{6\pi}\frac{A_0}{\delta^n} + \kappa\sigma. \qquad (17)$$

As the author is aware, expressions of only this type for the pressure are used for modelling the processes associated with evaporating films. The exponent n is mostly chosen to be $n = 3$, resulting in a continuous decrease of the disjoining pressure approaching asymptotically zero as the film thickness δ increases. Such a behaviour is indeed observed with very thin films of nonpolar liquids. However, experiments with polar liquids, like water, on different substrates indicate that the disjoining pressure disappears not asymptotically as δ tends to infinity but rather abruptly at finite values of the film thickness, somewhere between 2 nm and 8 nm, depending on temperature. Thereby a larger film thickness corresponds to a lower temperature (nearly 7.5 nm at 21°C, less than 2 nm at 50°C, both for water on mica [20]). This behaviour is attributed to the breakage of molecular structures formed at the liquid–wall interface which should be caused by thermal agitation of molecules. The disappearance of the disjoining pressure changes the evaporation conditions at the film surface essentially, because the rise of the saturation temperature due to the action of this pressure vanishes; also, the flow in the film may become disrupted. Therefore, the use of equation (17) in cases of polar liquids can lead to erroneous results.†

The equations given so far are sufficient to describe the flow in an evaporating film under the assumptions of the model. They are also valid for planar films, for which $\xi \to \infty$, and the equation (5) of continuity reduces to $\partial u_\xi/\partial \xi = 0$. These equations are basically the same as the well-known Nusselt equations for the laminar film condensation. This follows immediately from equation (11), which becomes the Nusselt equation for the velocity of the condensate if the pressure gradient is thought to be caused by the action of gravity, that is, if $\partial p_1/\partial \xi = -\rho_1 g$.

Before leaving this section, some general remarks seem appropriate regarding the validity of the above results. It should first be emphasised that the equations derived apply to a liquid film lying on an extremely smooth wall surface, and their use to describe the film flow along a rough surface can lead to serious errors. In all cases, in which the surface roughness is comparable to, or even larger than, the film thickness δ, the flow in the film will be three-dimensional and the hydrodynamic equations, deduced from the greatly simplified Navier–Stokes equations, including also equations (15) and (17) for the pressure distribution, become invalid, and the whole theory breaks down. The roughness of technical surfaces used for boiling experiments usually lies below 0.5 μm, which means that the above theory might become inapplicable in precisely the cases,

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† With polar liquids, both the structural and the molecular components of the disjoining pressure are observed and the corresponding pressure term in equation (17) should be replaced by $A_0/(6\pi\delta^n) + B/\delta^2$, with B depending on temperature and liquid properties [20].

where we would most like to apply it. Furthermore, the considerations rest on the tacit assumption that the classical hydrodynamics is unrestrictedly valid also for very thin liquid films. However, thin films may, and probably they do, behave differently with regard to their bulk phases due to the mutual interactions of the interfaces also in cases where the films separate two different phases [2], such as vapour in a growing bubble and solid wall.

3. HEAT FLUX

The sketched solution of the hydrodynamics of thin film needs knowledge of the flow rate Γ, see equation (16), if the film thickness δ should be determined. The quantity Γ must thus be chosen, in general as a function of the film thickness, or obtained along another way of reasoning. In systems with change of phase, say evaporation at the film surface, an energy balance is added to the hydrodynamic equations and the flow rate Γ is, so to say, replaced by the heat flux.

In the literature, e.g. ref. [5, 10], the energy balance is written for a film element, sandwiched between the two interfaces and extended by $d\xi$ along the coordinate ξ, as can easily be imagined on the basis of the sketches in Fig. 2. With q as the local heat flux at the wall surface, the energy balance, combined with the mass balance for the same control volume, gives

$$q = \left(\frac{\Gamma}{\xi} + \frac{d\Gamma}{d\xi}\right)\Delta h \qquad (18a)$$

where Δh denotes the latent heat (enthalpy) of evaporation.

Implementing the flow rate Γ according to equation (12) in equation (18a), we get

$$q = -\frac{1}{3}\frac{\Delta h}{v_1\xi}\frac{\partial}{\partial\xi}\left(\delta^3\frac{\partial p_1}{\partial\xi}\xi\right) \qquad (18b)$$

that is,

$$q = -\left(\frac{\delta^2\Delta h}{v_1}\frac{\partial p_1}{\partial\xi}\frac{d\delta}{d\xi} + \frac{1}{3}\frac{\delta^3\Delta h}{v_1}\left(\frac{1}{\xi}\frac{\partial p_1}{\partial\xi} + \frac{\partial^2 p_1}{\partial\xi^2}\right)\right).$$
$$(18c)$$

The different shapes of the expression for the heat flux are written down for requirements of comparisons, which shall be undertaken in the course of the paper. Equation (18a) or (18b) describes the profile of the free film surface as a function of the local heat flux; equations (18a–c) are based on the assumption of no change of the liquid temperature along the film flow. The liquid in the film is not allowed to absorb any heat and thus serves purely for heat transmission from the heated surface to the free film surface. The heat flux q within the film does, therefore, not change with the distance η from the wall. The expression given by

equation (18b) is mostly used in the papers dealing with evaporating films.

4. EXAMINATIONS OF THE FLOW AND HEAT FLUX MODELLING

4.1. General

The derivations in the previous chapters have been possible in the wake of several restrictions, which are therefore contained in the final expressions. Equation (5), for example, states that no axial flow in the film is allowed. This claim is incorporated into the expression for the radial velocity component u_ξ given by equation (11), and as such, it influences both the shape of the film surface and heat transfer. There are therefore serious reasons to ask for the consequences of the assumptions, and to clarify whether the crucial processes of bubble growth are sufficiently described by the simple model. Some of these consequences should be illuminated in the following.

4.2. Ignoring of the axial velocity component

To form an impression as to whether the axial liquid flow may legitimately be ignored, we shall determine the velocity u_η of this flow from the complete equation of continuity (3) using the expression (11) for the velocity u_ξ. This idea might appear questionable, inasmuch as the expression for u_ξ is obtained from the truncated equation of continuity. However, the only shortcoming of our working hypothesis is that u_η thus derived will be erroneous because u_ξ does not represent the actual velocity component in the film. To lend the idea more clarity, we may consider equation (11) to represent the true radial velocity component and ask then for the corresponding axial velocity forced by equation (3).

Achieving the goal requires putting equation (11) into equation (3) and remembering that $\partial p_1/\partial\xi \neq f(\eta)$; an integration then gives

$$u_\eta = -\frac{1}{\rho_1 v_1}\frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\left(\frac{\eta^3}{6} - \frac{\delta\eta^2}{2}\right)\frac{\partial p_1}{\partial\xi}\xi\right) \qquad (19a)$$

or

$$u_\eta = -\frac{1}{\rho_1 v_1}\left(\left(\frac{\eta^3}{6} - \frac{\delta\eta^2}{2}\right)\left(\frac{1}{\xi}\frac{\partial p_1}{\partial\xi} + \frac{\partial^2 p_1}{\partial\xi^2}\right) - \frac{\eta^2}{2}\frac{\partial p_1}{\partial\xi}\frac{\partial\delta}{\partial\xi}\right)$$
$$(19b)$$

if the condition on the wall ($\eta = 0$: $u_\eta = 0$) shall be satisfied.

The sign of the axial velocity u_η depends on the behaviour of the term

$$\left(\frac{\eta^3}{6} - \frac{\delta\eta^2}{2}\right)\frac{\partial p_1}{\partial\xi}\xi \sim (\eta - 3\delta)\eta^2\frac{\partial p_1}{\partial\xi}\xi$$

as ξ varies. For $\partial p_1/\partial\xi > 0$, this term is negative, and a decrease of its value along ξ would lead to a negative velocity u_η, that is, to a liquid flow towards the wall.

Applied to the interface, $\eta = \delta$, expression (19b) becomes

$$u_{\eta\delta} = \frac{\delta^2}{\rho_l \nu_l}\left(\left(\frac{1}{\xi}\frac{\partial p_l}{\partial \xi} + \frac{\partial^2 p_l}{\partial \xi^2}\right)\frac{\delta}{3} + \frac{1}{2}\frac{\partial p_l}{\partial \xi}\frac{\partial \delta}{\partial \xi}\right)$$

$$= \frac{1}{3}\frac{\delta^3}{\rho_l \nu_l}\frac{\partial p_l}{\partial \xi}\frac{\partial}{\partial \xi}\ln\left(\delta^{3/2}\frac{\partial p_l}{\partial \xi}\xi\right). \quad (20)$$

This equation is suitable to examine the simplification of the equation of continuity. For this purpose, we calculate the radial velocity $u_\xi = u_{\xi\delta}$ at the interface ($\eta = \delta$) from equation (11),

$$u_{\xi\delta} = -\frac{1}{2}\frac{\delta^2}{\rho_l \nu_l}\frac{\partial p_l}{\partial \xi} \quad (21)$$

and combine it with the axial velocity $u_{\eta\delta}$ to give

$$\frac{u_{\eta\delta}}{u_{\xi\delta}} = -\left(\frac{2}{3}\frac{\delta}{\xi} + \frac{\partial \delta}{\partial \xi} + \frac{2}{3}\frac{\partial^2 p_l/\partial \xi^2}{\partial p_l/\partial \xi}\delta\right). \quad (22)$$

In the ratio $u_{\eta\delta}/u_{\xi\delta}$, the velocity $u_{\xi\delta}$ appears as a measure of the axial liquid flow. This ratio is seen to depend only on the film thickness δ, which means, on the shape of the film profile, and, in general, the ratio will be larger the stronger the film thickness changes with ξ.

For a planar liquid film, we have $\xi \to \infty$, and the first term in the brackets of equation (22) disappears, whence

$$\frac{u_{\eta\delta}}{u_{\xi\delta}} = -\left(\frac{\partial \delta}{\partial \xi} + \frac{2}{3}\frac{\partial^2 p_l/\partial \xi^2}{\partial p_l/\partial \xi}\delta\right). \quad (23)$$

If this film has a constant thickness ($\delta = $ const) and is driven at a constant pressure gradient ($\partial p_l/\partial \xi = $ const), the ratio $u_{\eta\delta}/u_{\xi\delta}$ becomes zero, as expected.

Required for a numerical estimation of $u_{\eta\delta}/u_{\xi\delta}$ are the radial distributions of both the film thickness and pressure. Studies that provide such distributions are very rare to the literature. The results of numerical calculations performed by Stephan and Hammer [10, 11] seem to be the only ones existing at present, as far as a growing bubble is concerned. The authors assumed a liquid film beneath the growing vapour bubble and used the model equations as summarised above; the model conditions are briefly sketched in the introduction of the present paper. From the results thus obtained and presented graphically, it may be followed that, in a certain region of the so-called 'micro-zone' of the film close to the TPL ($\xi - \xi_{ad} > 0.15$ μm, ξ_{ad} being the radius of the non-evaporating layer and thus determining approximately the position of the TPL), the pressure p_l in the film changes almost linearly with the radial position ξ, giving $\partial^2 p_l/\partial \xi^2 \approx 0$. In this region, equation (22) may be simplified as

$$\frac{u_{\eta\delta}}{u_{\xi\delta}} \approx -\left(\frac{2}{3}\frac{\delta}{\xi} + \frac{\partial \delta}{\partial \xi}\right). \quad (24)$$

In the same film region, also the film thickness δ is, more or less, a linear function of ξ so that the values of δ and $\partial\delta/\partial\xi$, obtained with refrigerant R114, evaporating on a horizontal copper tube, are $\delta \approx 0.12$ μm and $\partial\delta/\partial\xi \approx 0.519$ at $\xi - \xi_{ad} \approx 0.29$ μm with $\xi_{ad} = 56.3$ μm (as can approximately be taken from Fig. 7.2 in ref. [10], or Fig. 4 in ref. [11]). Using these values, equation (24) gives

$$\frac{u_{\eta\delta}}{u_{\xi\delta}} \approx -0.52$$

which means that the velocities $u_{\xi\delta}$ and $u_{\eta\delta}$ are of the same order of magnitude; the axial velocity u_η contributes thus considerably to mass transport in the film. The negative sign indicates that one of the velocities has the opposite direction with respect to the system coordinates.

Equation (20) leads to an interesting relationship for the film thickness. With $u_{\eta\delta} = 0$, which is in accordance with the hydrodynamics of the models [9–13], it follows

$$\delta^{3/2}\frac{\partial p_l}{\partial \xi}\xi = \text{const} \quad (25)$$

which must give the same pressure distribution in the film as equation (17). In the region of the linear pressure distribution, $\partial p_l/\partial \xi = $ const and equation (25) yields

$$\delta = \frac{\text{const}}{\xi^{2/3}}. \quad (26)$$

The decrease of the film thickness δ as ξ increases, forced by equation (26), is clearly in contradiction to the basic idea of the model, see Fig. 2.

From our analysis presented so far, we may conclude that the ignoring of the axial liquid flow in the film is not justifiable and equation (11) for the radial liquid velocity is essentially erroneous. Considerations in the next section, however, will show that the axial liquid flow, though ignored within the film hydrodynamics, is accounted for in the energy balance, and the velocity $u_{\eta\delta}$ given by equation (19) is, in fact, contained in the expression for the heat flux.

4.3. Mass flow, heat flux and film profile

In the following, we will derive a simple relationship that connects the heat flux, liquid velocity, and shape of the film using elementary balances for matter and energy. On the basis of this relationship we will then examine the corresponding expression taken from the literature and discuss the curvature of the film surface at the TPL.

(a) Mass flow and heat flux

To keep the further considerations more illustrative, we first wish to define an elementary 'flow chan-

nel', Fig. 2(c). Such a channel is formed when the shaded area, located at an arbitrary distance η above the wall surface, is rotated round the axis η. The channel thus formed extends from the bubble surface towards the infinity; its cross-sectional area at an arbitrary radial position ξ is $2\pi\xi \, \mathrm{d}\eta$. If the corresponding fluid velocity is u_ξ, the mass flow in the channel is obtained from

$$\mathrm{d}\dot{M}_1 = 2\pi\rho_1\xi u_\xi \, \mathrm{d}\eta. \tag{27}$$

According to our key assumption, the axial velocity does not exist, $u_\eta = 0$, and, by equation (5), $\xi u_\xi = \text{const}$. Consequently, the mass flow $\mathrm{d}\dot{M}_1$ does not change along the flow path, the liquid flow entering the channel remains constant (as if the channel walls were impermeable) until the interface is reached. While passing the interface, the liquid evaporates, absorbing the heat $\mathrm{d}\dot{Q}_\delta$; at a constant latent heat of evaporation, the heat flow $\mathrm{d}\dot{Q}_\delta$ changes congruently with the mass flow $\mathrm{d}\dot{M}_1$. Since the velocity u_ξ increases with η, the same flow channel, located at a larger distance from the wall, conducts more liquid and thus requires a larger heat flow for evaporation. The reverse is true, when η decreases. This interrelationship will prove later to be decisive for a judgement about the curvature of the film surface at the TPL.

With the aid of our flow channel, we can very simply obtain an expression for the heat flux q_δ on the interface. Since $u_\eta = 0$, the energy balance $\mathrm{d}\dot{Q}_\delta = \mathrm{d}\dot{M}_1\Delta h$ gives immediately,

$$q_\delta = \rho_1 u_{\xi\delta}\Delta h \frac{\partial \eta}{\partial \xi}\bigg|_\delta = \rho_1 u_{\xi\delta}\Delta h \frac{\partial \delta}{\partial \xi} \tag{28a}$$

which, with $u_{\xi\delta}$ from equation (21), becomes

$$q_\delta = -\frac{1}{2}\frac{\delta^2\Delta h}{\nu_1}\frac{\partial p_1}{\partial \xi}\frac{\partial \delta}{\partial \xi}. \tag{28b}$$

The heat fluxes q_δ given by equation (28a) is related to the same area as the heat flux q according to equation (18a), namely $2\pi\xi \, \mathrm{d}\xi$. These equations, however, differ from each other. Already the first term on the right-hand side of equation (18c) is twice as large as the heat flux q_δ given by equation (28b). This is unexpected because both the equations are based on the assumption of no axial flow in the film and no change of the liquid temperature along the liquid flow. In this case, as mentioned above, the liquid not evaporating within the control volume serves purely for heat transmission from the heated wall to the film surface. The question why the equations disagree is important for us and shall be treated next.

An answer to this question is easily provided by a closer inspection of the energy balance that had led to equation (18b). The energy balance in question is written for a film element sandwiched between the wall and the film surface. At constant physical properties, it gives

$$q = \frac{\rho_1\Delta h}{\xi}\frac{\partial}{\partial \xi}\left(\int_0^{\delta(\xi)} \xi u_\xi \, \mathrm{d}\eta\right). \tag{29}$$

In papers dealing with film flow, the radial velocity u_ξ in equation (29) is replaced according to equation (11), the integration performed, and the derivation with respect to ξ conducted. In this way, equation (18b) is arrived at. Within the limits of the model assumptions, however, this is not the correct way because the boundary of integration, $\delta(\xi)$, depends on the same parameter ξ as the function to be integrated, and the derivative of the integral generally is not the same as the derivative of the function obtained by the integration. Otherwise, the basic assumption of the model is blasted, as shown immediately.

To treat equation (29) correctly, we have to apply the Leibnitz rule, giving

$$q = \frac{\rho_1\Delta h}{\xi}\left((\xi u_\xi)_{\delta(\xi)}\frac{\partial \delta}{\partial \xi} + \int_0^{\delta(\xi)}\frac{\partial}{\partial \xi}(\xi u_\xi) \, \mathrm{d}\eta\right). \tag{30}$$

The integral in equation (30) arises from the dependence of the film thickness $\delta = \delta(\xi)$ on the parameter ξ and represents a convective contribution to mass flow (and to heat flux). Its physical interpretation is obvious.

In order to perform the integration in equation (30), the function $\partial(\xi u_\xi)/\partial\xi$ needs to be specified, for which, within our analysis, two ways are generally possible. One of these can be based on the requirement of a strict fulfilment of the model assumption, by which no axial flow in the film does exist. Then, the derivative $\partial(\xi u_\xi)/\partial\xi$ is zero, as stated by equation (5), and equation (30) reduces simply to equation (28a). Clearly, along this line of reasoning, the model is kept consistent, and its conditions are nowhere violated.

On the other hand, we can leave the line of the model, ignoring the requirement by equation (5), and replace $\partial(\xi u_\xi)/\partial\xi$ according to the complete equation of continuity, equation (3). Then, equation (30) delivers

$$q = \frac{\rho_1\Delta h}{\xi}\frac{\partial}{\partial \xi}\left(\int_0^{\delta(\xi)}\xi u_\xi \, \mathrm{d}\eta\right) = \rho_1\Delta h\left(u_{\xi\delta}\frac{\partial \delta}{\partial \xi} - u_{\eta\delta}\right). \tag{31}$$

This expression for the heat flux is generally valid for two-dimensional films. It contains not only the radial but also the axial liquid velocity and, depending on these velocities, a variety of particular cases can easily be constructed. For example, if we insert equations (20) and (21) for the velocities $u_{\eta\delta}$ and $u_{\xi\delta}$ into equation (31), we get one of the equations (18). This example clearly shows that, notwithstanding the fact that the axial liquid flow is ignored within the hydrodynamics of the model, the heat flux calculated by equation (18) still depends on the axial flow velocity $u_{\eta\delta}$. Obviously,

we are led to a model inconsistency,† which, as the author is aware, has not been mentioned in earlier treatments.

This shortcoming of the models used in the literature answers immediately our question stated above in connection with the heat balances. Namely, denoting by Δq the difference of the heat fluxes q and q_δ given by equations (31), or one of equations (18), and (28a), we get

$$
\begin{aligned}
\Delta q &= q - q_\delta \\
&= -\rho_1 u_{n\delta} \Delta h \\
&= -\frac{1}{3} \frac{\Delta h \delta^3}{v_1} \frac{\partial p_1}{\partial \xi} \frac{\partial}{\partial \xi} \ln\left(\delta^{3/2} \frac{\partial p_1}{\partial \xi} \xi\right)
\end{aligned}
\tag{32}
$$

whereby equation (20) has been incorporated into the latter form of this expression. As equation (32) tells, the difference Δq is a convective contribution to heat transfer due to axial liquid flow, and it disappears only at the requirement of equation (25), which is the ultimate condition for vanishing of $u_{n\delta}$.

(b) Film profile

The film thickness δ is one of the key quantities in the relationships treated so far. It must therefore be possible to obtain, at least qualitatively, some features of this quantity on the basis of these relationships. In connection with evaporation during bubble growth, the change of the film thickness at the film tip is of particular interest for us. Thus, we should try next to arrive at clear and unequivocal conclusions regarding the film shape at the TPL, starting from equation (28a).

As noted above, equation (28a) is derived in accordance with the basic model assumption. The mass flow $d\dot{M}_1$, included in this expression, depends on the axial position η of the 'flow channel', and a larger value of

η corresponds to a larger mass flow requiring a larger heat flow for liquid to evaporate. Now, if the channel is located on the heated surface, as illustrated in Fig. 2(d), the liquid flow in this channel is expected to be very small because of the no slip condition on the wall surface. Consequently, the heat needed for evaporation of this flow will correspondingly be reduced.

This causality of the processes, obvious and without any objections within the framework of the hydrodynamics of the models, seems to contradict a simple reasoning. Namely, at the outlet of the channel placed on the wall, the liquid–vapour interface 'joins' the heated surface, and we may expect in this region a very large heat flux q_δ. A large heat flux corresponds to a high evaporation rate, which in the case of a motionless interface (and TPL) requires a large liquid velocity in the channel. However, due to wall proximity, the channel placed on the wall can hardly provide enough liquid to the motionless film tip necessary to maintain the large heat flux.

There is obviously some conflict between our reasoning and the model, something, which needs to be explained. A meaningful explanation should involve the film profile as a crucially interesting quantity. In order to simply understand and clarify the mutual interactions between the heat flux, fluid flow, and film profile, the following questions should guide our analysis:

—How does the system organise and adjust itself near the TPL?
—What happens at the film tip? Does the curvature change there unexpectedly?
—Is the TPL indeed motionless, or does it expand along the heated wall?

To examine these questions, we write equation (28a) as

$$
\frac{\partial \delta}{\partial \xi} = \frac{1}{\rho_1 \Delta h} \frac{q_\delta}{u_{\xi\delta}}
\tag{33}
$$

which shows that the slope $\partial \delta / \partial \xi$ of the film profile depends both on the physical properties (ρ_1, Δh) and process parameters (q_δ, $u_{\xi\delta}$). An increase in the velocity $u_{\xi\delta}$, for example, causes a smaller slope of the film surface if the other quantities are kept unchanged. On the contrary, at smaller values of $u_{\xi\delta}$, the film profile becomes steeper at a given heat flux. Obviously, in the region of the TPL, where the radial liquid velocity $u_{\xi\delta}$ is practically zero, the slope $\partial \delta / \partial \xi$ is very large, even at a low heat flux. In the limiting case $u_{\xi\delta} \rightarrow 0$ (or $q_\delta \rightarrow \infty$), we get $\partial \delta / \partial \xi \rightarrow \infty$, and the dynamic (micro) contact angle at the film tip tends to $\pi/2$.

Farther away from the TPL, the film thickness δ increases, resulting in a larger velocity $u_{\xi\delta}$ and possibly in a smaller heat flux q_δ. Therefore, in the light of equation (33), the derivative $\partial \delta / \partial \xi$ is expected to reduce as ξ increases. The other limiting value for the

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† The hydrodynamic model inconsistency has further consequences. Namely, since equation (18b) accounts for the axial liquid flow, the temperature ϑ in the film does not change linearly with the wall distance and the use of the classical Fourier relationship for calculating the temperature at the film surface and the heat transfer coefficient can result in considerable errors. The heat flux q in the film is generally not constant but changes according to

$$
q = -\lambda_1 \frac{\partial \vartheta}{\partial \eta} + \rho_1 c_{p1} u_\eta \vartheta
$$

where λ is the thermal conductivity and c_p the specific heat capacity of the liquid. However, the most important effect of the axial liquid flow lies possibly in the radial temperature change which makes the validity of equation (18b) seriously questionable.

On the contrary, if we—in agreement with literature—assume equation (18b) to represent the true heat flux, then the actual velocity components were given by equations (11) and (19a). In this case, the continuity equation (3) were satisfied, but not the momentum equations. Clearly, seen on the basis of such an assumption, we were not able to form a set of consistent equations that govern the transport processes in the film.

slope is thus, $\partial\delta/\partial\xi \to 0$, giving a film surface parallel to the wall surface (see Fig. 1).

On the basis of equation (33), the following conclusion seems natural: due to the negligibly small radial velocity $u_{\xi\delta}$ in the ultra-thin liquid layer immediately adjacent to the heated wall, the evaporation at the TPL generates a convex curvature of the interface at the tip of the liquid wedge. This is likely to occur already at a low heat flux, as was suggested by the author earlier [15, 16].

Also Straub [21] was recently led to similar results on the basis of his explanations of heat transfer and evaporation at the TPL. The experiments by Wayner et al. [22, 23] seem to support the above conclusions. These authors studied evaporation at liquid menisci, formed on a horizontal plate-heater and reported, among other things, about the bending down of the film profile. The term 'bending down' characterises the change of the curvature of the film surface in the region of the TPL when heated in comparison to the curvature without heating; this finding is qualitatively in accordance with equation (33).

The above reasoning and conclusions deduced from equation (33) are beyond any doubt, notwithstanding the fact that the energy balance, upon which this equation is based, is incorrect so far as it does not account for any axial liquid flow. This lack, however, can simply be eliminated if the derivative $\partial\delta/\partial\xi$ is obtained from equation (31), thus,

$$\frac{\partial\delta}{\partial\xi} = \frac{1}{\rho_l\Delta h}\frac{q_\delta}{u_{\xi\delta}} + \frac{u_{\eta\delta}}{u_{\xi\delta}}. \qquad (34)$$

The term $u_{\eta\delta}/u_{\xi\delta}$ does by no means alter our conclusions.

Some of the questions stated above can now be answered by emphasising that a strong change of the curvature of the film tip is unavoidable if at the motionless TPL a high heat flux should be established. This answer seems to be the most probable one within the model assumptions. However, in a system with more freedom, a one less restricted by our assumptions, other states and processes, not mentioned so far, are possible. The system could, for example, generate a liquid flow along the interface towards the TPL. For such a flow to occur by the classical Marangoni convection in single-component systems, the temperature of the film surface is required to decrease as the TPL is approaching. At first glance, this requirement might sound inconceivable. However, a decrease of the temperature while approaching the TPL may indeed be expected because of the convex curvature at the film tip. Convex curvature requires a lower equilibrium temperature thus leading to a lower actual temperature in that region. Another possibility is a continuous movement of the TPL radially outwards. This notion is best supported by the fact that the liquid layer adjacent to the heated surface is immobile, and the TPL will, therefore, 'slide' towards the liquid bulk as the evaporation proceeds and the bubble grows.

Also the influence of physical properties $(\rho_l, \Delta h)$ appearing explicitly in equations (33) and (34) on the film profile is of interest for several reasons and should shortly attract our attention. These equations indicate a stronger change of $\partial\delta/\partial\xi$, resulting in a steeper film profile, as the pressure increases. Near the thermodynamical critical point of the fluid, we may expect a specific situation, because $\rho_l\Delta h \to 0$, and $\partial\delta/\partial\xi \to \infty$, no matter how low the heat flux $q_\delta > 0$ may be. This causality might be of some importance with regard to burn-out heat flux, which reduces in the range of higher pressure as the pressure increases, approaching zero at the thermodynamical critical point. This fact could also give some indications how to understand the formation of jet-like two-phase structures observed, when boiling occurs in the near-critical fluid region. In connection with the film shape at the motionless TPL, we should further emphasise that— even at an extremely large heat flux—the maximum micro-contact angle does not exceed $\pi/2$ (at $\partial\delta/\partial\xi \to \infty$). This follows immediately from equation (33). Obviously, the models, which consider the TPL to be motionless, do not allow contact angles larger than $\pi/2$. Such models do, therefore, not indicate the mode of transition or film boiling to exist.

Before closing the curvature considerations, we should remark that our results are valid within the continuum fluid domain. The considerations suffice, therefore, to enter the region of the TPL, but not to reach the TPL itself, the treatment of which belongs to the domain of molecular dynamics.

4.4. Liquid flow in the wedge

The direction of liquid flow in the film, as assumed in the models [9–13], is unquestionable as long as the free film surface is actually motionless and the axial velocity is zero. Such a physical picture, however, can hardly be expected in the case of a growing vapour bubble, where the whole process is strongly time dependent and the interface not motionless. Due to rapid evaporation at the TPL, the curvature of the film surface changes both in time and space. Depending on heat flux, the interface assumes a more or less pronounced convex curvature at the film tip giving rise to a corresponding Laplace pressure, Fig. 1. If, as an example, at the tip of a two hundred nanometers thick liquid film, in which the disjoining pressure might reduce almost to zero, a radius of the convex curvature of $r = 100$ nm is assumed (concave curvature is put at zero), a Laplace pressure of $\sigma/r = 10^5$ N m^{-2} for a fluid having a surface tension of $\sigma = 10^{-2}$ N/m (typical value for refrigerants) is expected. This pressure is acting upon the film tip and is likely to push the liquid radially outwards during the main period of bubble growth, Fig. 3(a). The pushing of the liquid does in no way hinder the evaporation. In contrast, a convex curved interface requires a larger vapour pressure, thus facilitating the evaporation. The co-operative action of the Laplace pressure and evaporation at the TPL results in a continuously 'sliding'
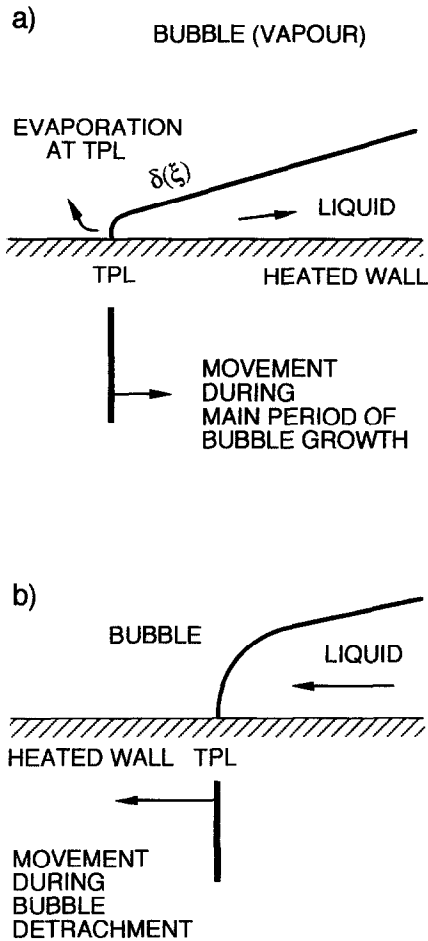
a)

BUBBLE (VAPOUR)

b)

Fig. 3. Direction of flow in the liquid wedge established along the rim of a growing vapour bubble. (a) Cooperative action of evaporation at the TPL and Laplace pressure causes a pushing of the liquid radially outwards during the main growth period. (b) The TPL and the liquid rush radially inwards during the bubble detachment process. The bubble takes off, when the concave radius of the interface at the film tip becomes zero.

of the TPL along the heated surface and, during the first period of bubble growth, there seems to occur no processes whose action could stop this sliding; thus, any *ad hoc* assumption of an inwards flow in the liquid film seems unjustifiable. The physical picture just sketched is not altered by a possible adsorption on the heated surface, as discussed elsewhere [24]. After a certain period of time, the outwards sliding of TPL becomes interrupted, then started again, but now in the reverse direction, Fig. 3(b). This instant fluxes the time origin of bubble detachment process.

The sliding of the TPL is governed by the processes of heat transfer and fluid flow in the very narrow region around the TPL. The most important contribution to this heat transfer stems probably from the heated wall. As discussed in ref. [16], due to the strong temperature drop of the heated surface at the position of the TPL and the faster propagation of temperature

waves in the solid wall than in the liquid, the heat, accumulated (prior to bubble generation) in the thin liquid layer adhering to the heated surface, can now become re-absorbed by the wall and then, by conduction, reach the heat sink acting along the TPL. The possibility of a reversal of the heat flow at a particular distance from the TPL depends on transport properties of both liquid and heated wall as well as on the wall superheating.

The sliding velocity of the TPL could roughly be estimated on a supposition of a motionless liquid in the film. In such a case, the sliding of the TPL would be caused by evaporation only and its velocity would approximately be the same as the velocity of the liquid at the fixed TPL. Thus, with $u_{\mathrm{TPL}} = u_{\xi\delta}$ equation (28a) gives

$$u_{\mathrm{TPL}} = \frac{q_\delta}{\rho_1 \Delta h \partial \delta / \partial \xi}. \tag{35}$$

This equation is invalid for $\partial \delta / \partial \xi \to \infty$ because the elementary area of the wall surface transferring the heat q_δ would reduce to a line (TPL) and the heat balance, upon which the equation (35) rests, would lose its physical sense. At $\partial \delta / \partial \xi \to \infty$, the velocity u_{TPL} could roughly be estimated from

$$u_{\mathrm{TPL}} = \frac{q_1}{\rho_1 \Delta h} \tag{36}$$

where q_1 is the heat flux at the vapour–liquid interface in the vicinity of the TPL, and not at the wall surface. Note that q_δ and q_1 in equations (35) and (36) must account for the momentum change due to evaporation and the action of the Laplace pressure if the equations should approximately give the actual velocity values. In other words, the velocity u_{TPL} has to be calculated from the complete momentum equation.

To obtain an estimate of the velocity u_{TPL} by equation (36) and to illustrate the sliding of the TPL, we choose $q_1 = 10^7 \mathrm{W\,m^{-2}}, \rho_1 = 10^3 \mathrm{kg\,m^{-3}}$, and $\Delta h = 10^5$ J kg^{-1}, giving $u_{\mathrm{TPL}} = 0.1$ m s^{-1}. Assuming now a radial shifting of TPL during the bubble growth to be 100 μm (radial distance reported by Hammer [10] is approximately $\xi_{\mathrm{ad}} \approx 60\ \mu$m), the time period required for the TPL to move along this distance would be $\Delta \tau = 10^{-3}$ s, which is an acceptable order of magnitude in comparison with typical times of bubble growth. Note that larger values of u_{TPL} and smaller ones of $\Delta \tau$ are expected at larger pressures because the term $\rho_1 \Delta h$ decreases as the pressure rises.

4.5. Boundary condition at the film surface

We turn now to consider the boundary condition $(\eta = \delta : \partial u_\xi / \partial \eta = 0)$ at the film surface chosen for determination of the radial velocity u_ξ. This condition, however suggestive as it is, seems unlikely to establish itself in reality. Particularly in the case of a fast growing bubble, the stress field at the interface changes very rapidly with time. In addition, it is a complex function of several parameters, including gradient of

surface tension and momentum change (reactive force) due to evaporation. For the condition to be fulfilled, the Newton shear stress, $v_1\rho_1\,\partial u_\xi/\partial\eta$, must be zero at $\eta = \delta$, at each instant. This requires the sum of the components (excepting the Newton term) of the stress tensor at the film surface to disappear, when projected on the plane $\eta = \text{const}$, during the whole period of bubble growth.

To illustrate why the boundary condition seems improbable also at steady-states, we first consider the velocity profile in the film at an arbitrary radial distance ξ, Fig. 2(b) and (c). At a distance $\eta < \delta$, the velocity of a thin liquid layer ('flow channel') at $\eta + d\eta$ is larger than at $\eta - d\eta$, that is $\partial u_\xi/\partial\eta \neq 0$, and the shear stress does exist. At $\eta = \text{const}$, the derivative $\partial u_\xi/\partial\eta$ changes along the flow path, but it still remains different from zero. In order to get $\partial u_\xi/\partial\eta = 0$ and to satisfy the boundary condition, the liquid layer at $\eta + d\eta$ must be decelerated, while at $\eta - d\eta$ accelerated. This, however, would change the mass flow in the channel, something, which is impossible within the framework of the model.

Similar conclusions follows immediately from equation (5). Applied to the film surface, this equation becomes

$$\frac{u_{\xi\delta}}{\xi} = -\left.\frac{\partial u_\xi}{\partial\xi}\right|_\delta = -\left.\frac{\partial u_\xi}{\partial\eta}\right|_\delta \frac{\partial\delta}{\partial\xi}. \tag{37}$$

Normally, $u_{\xi\delta}/\xi \neq 0$ and $\partial\delta/\partial\xi \neq \infty$. The derivative $\partial u_\xi/\partial\eta$ at the film surface is, therefore, different from zero. Only in the limiting case $\partial\delta/\partial\xi \to \infty$ (region of the TPL, as discussed above), $\partial u_\xi/\partial\eta$ would be zero. In such a case, however, the physical model would lose its original concept of representing a liquid film whose thickness continuously changes at a finite rate.

Another was to examine the boundary condition can be taken from a paper of Lord Rayleigh [25] published in 1890. While discussing the equilibrium conditions at the TPL (Young-equation), Rayleigh noted that a free surface of a wedge-shaped liquid film (actually the rim of a sessile droplet) can only be at equilibrium if a suitably defined potential along it is constant. Otherwise the gradient of the potential would give rise to a net force between adjacent elements and cause them to move. As to the suitably defined potential in our case, it suffices to note that the first term in equation (17) stems even from a potential at the film surface. Since the pressure along the film surface is not constant, but changes as $\partial p_1/\partial\delta = (\partial p_1/\partial\xi)/(\partial\delta/\partial\xi)$, the potential changes too, resulting in a stress in the plane $\eta = \text{const}$.

In the face of this analysis, we may conclude that $\partial u_\xi/\partial\eta$ is not generally expected to disappear at the interface, and another boundary condition should be asked for. As an alternative, one could specify the velocity at the surface, $\eta = \delta : u_\xi = u_{\xi\delta}$, in which case, the expression

$$u_\xi = u_{\xi\delta}\frac{\eta}{\delta} - \frac{1}{2}\frac{\delta\eta}{v_1\rho_1}\left(1-\frac{\eta}{\delta}\right)\frac{\partial p_1}{\partial\xi} \tag{38}$$

for the radial velocity is obtained, instead of equation (11). Note that equation (38) does not make the model more complex in comparison to equation (11) and the boundary condition, upon which it rests, is unquestionably always satisfied.

4.6. Some additional remarks
  (a) Liquid viscosity and nucleate boiling

The foregoing considerations have shown that the motionless TPL and the inwards liquid flow are improbable. Even the reverse seems true during the main period of bubble growth; both the TPL and the liquid in the film move radially outwards. However, since the movement of the TPL can be thought to be stopped and the liquid to flow correspondingly, as adopted in the models [9–13], one could indeed expect from the models in question to give an adequate description of heat transfer. The results obtained along this line of reasoning would not be satisfying. The main reason for our doubt lies in the action of liquid viscosity. For a fixed interface, the liquid viscosity would dominate the flow in the liquid wedge and, therefore, largely govern the bubble growth and boiling heat transfer. This is by far not the case.

The influence of the viscosity on nucleate boiling heat transfer seems to be a very weak one, if any, as far as the commonly viscous liquids are concerned. In fact, the most heat transfer correlations, based on experiments alone, do not suggest the liquid viscosity to belong to the family of decisive parameters. The weak effect of the viscosity, reported in some correlations, e.g. ref. [26], might be ascribed to liquid convection in bubble-free regions of heated surfaces and to external flow originated by bubble growth and detachment. Certain difficulty arises only with refrigerants inasmuch as a processing of numerous data, obtained with these fluids, indicates the liquid viscosity to facilitate the boiling heat transfer [27]. This is in contradiction to equations for nucleate boiling that root in models of single phase heat transfer and that mostly adopt the influence of the viscosity from these models, see, for example, discussion in ref. [28]. However, both the correlation for heat transfer with refrigerants and the model equations are irrelevant for the present considerations, not only because of their mutual contradictions with regard to liquid viscosity.

The results of a recent experimental work by Ammerman et al. [29] are of more importantance for our purpose. The authors found with FC-72 as working fluid the transport of latent heat to be the dominating mechanism in the developed boiling region. Now, the parameters that govern the latent heat transport are practically those of bubble growth and detachment, and the relationships describing these processes in the thermally controlled growth region do not contain liquid viscosity. This interchange clearly support our notion upon the viscosity effect. However, in spite of this fact, it would probably be too simple to follow

from these observations that the influence of liquid viscosity on nucleate boiling is entirely negligible. Because the physical properties of saturated liquids are linked to each other via temperature, the liquid viscosity might implicitly affect the bubble processes. In addition, there are some model expressions for bubble growth in the inertia controlled growth region that explicitly show a weak viscosity effect, see e.g. ref. [28].

(b) Surface roughness and sliding of the TPL

The processes taking place at the TPL are essential for bubble growth and latent heat transfer. Consequently, the length of the TPL must have some significance in the whole process of nucleate boiling, and a larger length of this line is expected to improve the heat transfer. Since the actual length of the TPL depends on micro-relief of the heated surface, the surface roughness influences the boiling process not only by bubble nucleation and bubble density, but also by the length of the TPL.

In connection with the roughness effect, the movement of the TPL should also be mentioned. The sliding of the TPL is accompanied by instantaneous, jumpwise change of its length. Namely, as the TPL slides continuously in the plane of the heated surface, it climbs up and surmounts the roughness elements thus becoming suddenly (jump-wise) longer. This rises the evaporation rate, resulting in an emission of pressure waves into the vapour space. The properties of the waves are largely determined by the properties of the ridges that emit them. The waves, not dissipated within the vapour phase, reach and interact with the bubble surface, most probably in a 'soft' way. The term soft should mean that the waves become not reflected from, but absorbed at the interface due to vapour condensation. The waves serve therefore as a means for transport of matter and heat at the sound speed in the vapour phase.

It should be noted that each micro-element of the surface roughness will emit, so to speak, its 'individual' wave. Corresponding to the large variety of these elements, a broad spectrum of the pressure waves may be expected. While travelling through the vapour, the waves interact with one another leading to a corresponding change of their properties responsible for heat transport. To quantify this transport requires a detailed analysis of generation and mutual interaction of the waves, including also the surface relief, which seems hardly possible at the time.

(c) Momentum pressure

The counteracting process regarding the rapid evaporation at the TPL in a given system seems to be the evaporation itself, originating a liquid acceleration by bubble growth. Such an acceleration roots in the momentum change of the evaporating liquid, which results in a dynamical pressure jump at the vapour–liquid interface by approximately $\Delta p(q_l/\Delta h)^2/(\rho_v\rho_l) \approx$

$(q_l/\Delta h)^2/\rho_v$, as it follows from a momentum balance for a plane, motionless interface. This pressure jump is a sensitive function of system pressure; it can reach relatively large values, particularly in the range of low vapour density and must be surmounted by a corresponding rise of the interfacial temperature.

It might be illustrative to estimate the dynamical pressure jump. If we, for example, take again the above used values of q_l and Δh and assume the vapour density to be $\rho_v = 1$ kg m^{-3}, we obtain $(q_l/\Delta h)^2/\rho_v = 0.1$ bar. Our illustration indicates that the dynamical effects associated with large evaporation rate at the TPL during bubble growth should be accounted for in a detailed analysis of heat transfer.

5. CONCLUSIONS

Recent models of heat transfer associated with bubble growth are resting on the notion of a liquid film that should form beneath the growing bubble. The inner part of the film is considered as an adsorption layer, incapable of evaporation, while the outer part of the film is vapour-generating. The film surface is only concave curved and is assumed to be either fixed in space [9], or to move jumpwise radially outwards [10–13], as the bubble grows. This movement results in an increase of the area covered with the non-evaporating film. In order to make the evaporation possible, in both models the liquid in the evaporating part of the film is assumed to flow radially inwards at steady-state.

In the present paper, we analysed some assumptions adopted in the models and showed that there is a great deal of diversity 'around and within' the growing bubble. For example, the liquid velocity in the evaporating film which is orthogonal to the heated surface is ignored within the hydrodynamics of the models. Our analysis indicates that the neglecting of this velocity component can result in too large errors when calculating the film properties. Furthermore, the boundary condition at the film surface (no shear stress) adopted in the models for determination of the liquid flow in the film has been found in this paper to be unlikely, particularly in the case of a fast growing bubble; such a condition is impossible within the basic model assumptions. In addition, the models are shown to be inconsistent since the axial velocity (orthogonal to heated wall) in the film is ignored within the film hydrodynamics, but at the same time included into the energy balance. This gives a non-linear distribution of the temperature in the film which is also in contradiction to the basic model. There are no physical reasons that could justify these facts. As the author is aware, the inconsistencies have not been realised in earlier treatments.

Further considerations in the present paper based on elementary balances for matter and energy indicate that the bubble interface in the region, where the three phases (vapour, liquid, solid) meet and act on each other, the region of the three-phase-line (TPL), is con-

cave–convex. This is because of the rapid evaporation at the tip of the liquid wedge formed along the rim of the growing bubble. The ultra-thin liquid layer of this wedge that is immediately adjacent to the heated surface is practically motionless due to no slip boundary condition on the wall surface. This layer is, therefore, incapable of supplying the quantity of liquid to the wedge tip necessary for evaporation to occur rapidly at a fixed TPL. Consequently, the TPL is not fixed; it moves 'towards' the liquid bulk during the main period of bubble growth. This movement is governed by the co-operative action of evaporation at the TPL and the Laplace pressure that pushes the liquid in the wedge radially outwards. The inwards movement of the liquid occurs during the constriction of the area enveloped by the TPL which precedes the bubble detachment.

It should be stressed that our considerations and the results we arrived at are resting on very simple relationships and the more complete balance-equations, once solved, will show at which places the conclusions are incomplete or invalid.

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